EUCLIDEAN FIELD THEORY AND SINGULAR CLASSICAL FIELD CONFIGURATIONS

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Abstract

Euclidean field theory on four dimensional sphere is suggested for the study of high energy multiparticle production. The singular classical field configurations are found in scalar ϕ^4 and SU(2) gauge theories and the cross section of $2 \rightarrow n$ is calculated. It is shown, that the cross section has a maximum at the energy compared to the sphaleron mass.

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One of the most interest topics of the last years is that of high energy multiparticle production, in which the final many-particle state can be described as a semiclassical one and when the classical solutions play an important role (examples of such processes are the electroweak processes, accompanied by baryon number violation[1-6], multijet production in strong interactions[7,8]). By investigating these questions we come to the calculational problem of dealing with $2 \to n$ processes with the final state considered semiclassically, but the initial state as a quantum state. Describing the final state as a semiclassical with definite energy and going back in time to the initial two-particle state, one violates the energy conservation low. This means that we have to consider the transition between states with the deferent energies. A way to circumvent this difficulty is suggested in the consideration of the singular classical trajectories in the imaginary (Euclidean) time, which were introduced by Landau 60 years ago in the calculation of the transition probability between the low and high energy quantum mechanical states[9]. This approach has been generalized to the quantum field theory by Iordanskji and Pitaevskji[10]. On the basis of their approach S.Yu. Khlebnikov[11] has suggested the use of singular Euclidean solutions for study of the above mentioned problem. The result of Landau has been used by M.Voloshin[12] in four dimensional ϕ^4 theory by considering spatially constant fields. Recently D.Diakonov and V.Petrov[13] developed this approach for the double-well potential and applied to the Yang-Mills theory. They obtained some approximate singular solutions in pure Yang-Mills and elektroweek theories. At low energies their results coincide with those of instanton-induced multiparticle production cross section, but at higher energies they yield exponentially decreasing behavior of cross section in the running gauge coupling constant. The cross section has a maximum at the energy defined by sphaleron mass.

1. EUCLIDEAN FIELD THEORY APPROACH ON 4^4.

We propose below the study of above mentioned problem by the method based on a field theoretical approach, which has been developed in [14]. In particular it has been suggested an Euclidean field theory on sphere S^4 and has been shown, that the system evolves along the radius of S^4. The dilatation operator is the evolution operator. This approach is convenient especially in scale invariant theories, in which the dilatation operator can be diagonalized. We recapitulate briefly some salient features of this approach. Consider for simplicity the real scalar massless field $\phi(x)$ in Euclidean space. Following[14] we introduce the spherical coordinates:

$$x_\mu = r\alpha_\mu(\vartheta, \varphi, \psi), \tag{1}$$

where $\alpha_\mu(\theta, \varphi, \psi)$ is a unit vector. Defining new field $\chi(r, \alpha) = r\phi(r, \alpha)$ the action is written

$$S = \frac{1}{2}\int_0^\infty \frac{dr}{r}\int d\alpha\Big\{(r\frac{\partial\chi}{\partial r})^2 + \chi(\hat{L}^2 + 1)\chi\Big\}, \tag{2}$$

where $\hat{L}^2$ is a square of angular momentum. For field $\chi(r, \alpha)$ the following commutation relations hold:

$$[\chi(r, \alpha), \chi(r, \alpha')] = [\dot{\chi}(r, \alpha), \dot{\chi}(r, \alpha')] = 0, \qquad [\dot{\chi}(r, \alpha), \chi(r, \alpha')] = -\delta^3(\alpha - \alpha'), \tag{3}$$

Here we introduced the notation: $\dot{\chi}(r,\alpha) = \partial\chi(r,\alpha)/\partial(\ln r)$ and the δ function is defined on the sphere:

$$\int d\alpha_1 f(\alpha_1)\delta^3(\alpha_1 - \alpha_2) = f(\alpha_2). \tag{4}$$

The nonvanishing commutator becomes usefull, if we introduce the proper time $\tau = -i\ln r$. Really

$$[\dot{\chi}(\tau,\alpha), \chi(\tau,\alpha')] = -i\delta^3(\alpha - \alpha'), \tag{5}$$

which is formally similar to the equal-time commutator of conventional field theory. The equation of motion for field $\chi(r,\alpha)$ is

$$\frac{\partial^2\chi(r,\alpha)}{\partial(\ln r)^2} - (\hat{L}^2 + 1)\chi(r,\alpha) = 0 \tag{6}$$

allows the separation of the variables. The eigenfunctions of the angular operator $\hat{L}^2$ form a complete orthonormal set of spherical functions $Y_{lnm}(\vartheta,\varphi,\psi)$:

$$\hat{L}^2 Y_{lnm}(\vartheta,\varphi,\psi) = l(l+2)Y_{lnm}(\vartheta,\varphi,\psi) \tag{7}$$

and are given by

$$Y_{lnm}(\vartheta,\varphi,\psi) = N_{lnm}e^{im\varphi}\sin^n\vartheta G^{n+1}_{l-n}(\cos\vartheta)\sin^m\psi G^{m+1/2}_{n-m}(\sin\psi), \tag{8}$$

where N_{lnm} is a normalization constant and $G^m_n(x)$ is a Gegenbauer polynomial. The numbers $l, n, m,$ are integers:

$$l = 0,1,2,..., \qquad n = 0,...l, \qquad m = -n,...n. \tag{9}$$

Taking into account the radial part of $\chi(\tau,\alpha)$ and using the proper time τ one can write the following expansion for $\chi(\tau,\alpha)$:

$$\chi(\tau,\alpha) = \sum_{l=0}^{\infty}\sum_{n=0}^{l}\sum_{m=-n}^{n}[a^{(-)}{}_{lnm}\frac{e^{-i\tau(l+1)}}{2l+2}Y^*_{lnm}(\alpha) + a^{(+)}{}_{lnm}\frac{e^{i\tau(l+1)}}{2l+2}Y_{lnm}(\alpha)]. \tag{10}$$

The hermiticity condition applied to $\chi(\tau,\alpha)$ shows, that $\chi(\tau,\alpha)$ is Hermitian for real τ and in this case with $[a^{(-)}_{lnm}]^+ = a^{(+)}_{lnm}$. Considering $a^{(-)}{}_{lnm}$ as an annihilation operator, we define the vacuum state $|0>$ as being annihilated by all $a^{(-)}{}_{lnm}$ operators:

$$a^{(-)}{}_{lnm}|0>= 0.$$

Next one can compute the vacuum expectation value of the product of two operators. In this way the following remarkable expression for the propagator $D(x_1 - x_2)$ is obtained: $(\Box D(x_1 - x_2) = -\delta^4(x_1 - x_2))$:

$$D(x_1 - x_2) = \frac{1}{4\pi^2|x_1 - x_2|} = \frac{1}{2\pi^2}\int_{-\infty}^{\infty} d\varepsilon \sum_{lnm} \frac{F^*{}_{lnm}(x_1)F_{lnm}(x_2)}{\varepsilon^2 + (l+1)^2}. \tag{11}$$

The functions $F_{lnm}(x) = r^{i\varepsilon-1}Y_{lnm}(\alpha)$ define the transformation between the Euclidean coordinate space and the conjugate space (εlnm), in which the dimensionality ε and the quantum numbers l, n, m result from diagonalization of their respective dilatation and angular momentum operators. In this space, in particular for scale invariant theory the propagator is diagonal:

$$D(\varepsilon, l, n, m) = \frac{1}{\varepsilon^2 + (l+1)^2} \tag{12}$$

and has poles at the eigenvalues of the evolution operator.

Next we consider the interaction with the external source. The original equation of motion in Euclidean coordinates reads:

$$\Box\phi(x) = \eta(x). \tag{13}$$

One may expand $\chi(\tau, \alpha)$ as follows:

$$\chi(\tau,\alpha) = \sum_{l=0}^{\infty}\sum_{n=0}^{l}\sum_{m=-n}^{n}[A^{(-)}{}_{lnm}(\tau)\frac{e^{-i\tau(l+1)}}{2l+2}Y^*{}_{lnm}(\alpha)+A^{(+)}{}_{lnm}(\tau)\frac{e^{i\tau(l+1)}}{2l+2}Y_{lnm}(\alpha)], \tag{14}$$

so that, the quantities $A^(\pm)_{lnm}(\tau)$ obey the following equation:

$$\frac{dA^{(\pm)}{}_{lnm}(\tau)}{d\tau} = \pm(l+1)A^{(\pm)}{}_{lnm}(\tau) \pm \frac{(-1)^{(m\pm m)/2}}{2l+2}\eta^{\pm}{}_{lnm}(\tau), \tag{15}$$

where $\eta^{\pm}_{lnm}(\tau)$ is a (εlnm) transform of source function. Introducing the evolution operator one can show, that it is defined by

$$U(r, r_0) = R\exp\{-i\int_{r_0}^{r} d(\ln r')D^{int}(r')\}, \tag{16}$$

where R denotes the ordering along the radius of S^4, and $D^{int}(r)$ is an interaction part of dilatation operator. The integration is over all space bounded by the spheres with $r_0^2 < r^2 < r^2$. It will be noted, that the scale invariance of the theory is not necessary requirement. In that case dilatation operator is still an evolution operator, but can not be diagonalized. In principle all scale-breaking terms can be treated perturbativly.

Comparing this approach with the conventional theory constructed on $t = const$ surface we see, that the energy and 3-momentum are replaced by dimensionality and the numbers (lnm). So in considering the multiparticle production iniciated by annihilation of two particles we describe the initial and final states by dimensionality ε instead of energy. In order to obtain the physical on shell amplitude we have to calculate first the two-point Green function, take its transform in (εlnm) space (instead of Furier transform in conventional theory) and then apply the procedure of LSZ. In this way we connect the probability of reaction with the full Green function.

2. LSZ FORMALISM.

We recall now briefly the LSZ reduction formalism in the context of field theoretical approach suggested in[13]. For simplicity we consider a scalar massless field. Let us postulate first $|in>$ and $|out>$ states as the asymptotic states of the interacting field in the limits $\tau \to -\infty$ and $\tau \to \infty$ respectively. These asymptotic fields $\chi_{in}(\tau,\alpha)$ and $\chi_{out}(\tau,\alpha)$ satisfy free equation of motion:

$$\hat{\mathbf{A}}(\tau,\alpha)\chi_{\substack{in\\out}}(\tau,\alpha) \equiv \Big(\frac{\partial^2}{\partial\tau^2} + (\hat{L}^2(\alpha)+1)\Big)\chi_{\substack{in\\out}}(\tau,\alpha) = 0$$

We define the following "time"-independent scalar product of two functions:

$$<\phi_1,\phi_2>= \int d\alpha\phi_1 \frac{\overleftrightarrow{\partial}}{\partial\tau}\phi_2. \tag{17}$$

For fields $\chi_{\substack{in\\out}}(\tau,\alpha)$ the expansion (10) holds. The creation operator $a^+_{\substack{in\\out}}(lnm)$ is expressed through $\chi_{\substack{in\\out}}(\tau\alpha)$ as follows:

$$a^+_{\substack{in\\out}}(lnm) = -i\int d\alpha e^{-i\varepsilon\tau}Y_{lnm}(\alpha)\frac{\overleftrightarrow{\partial}}{\partial\tau}\chi_{\substack{in\\out}}(\tau\alpha).$$

Let us denote the complete set of (εlnm) through q and consider the transition amplitude $< q'{}_1,\ldots,out|q_1,\ldots,in>$

$$\begin{aligned} &< q'_1,\ldots,out|q_1,\ldots,in>=< q'_1,\ldots,out|a^+_{in}|q_2,\ldots,in>= \\ &-i\int d\alpha e^{-i\varepsilon\tau}Y_{lnm}(\alpha)\frac{\overleftrightarrow{\partial}}{\partial\tau_1} < q'_1,\ldots,out|\chi_{in}(\tau_1,\alpha)|q_2,\ldots,in>. \end{aligned} \tag{18}$$

Since the integral is independent of τ_1

$$\begin{aligned} &< q'_1,\ldots,out|q_1,\ldots,in>= \\ &-i\lim_{\tau_1\to-\infty}\int d\alpha e^{-i\varepsilon\tau}Y^*_{lnm}(\alpha)\tfrac{\overleftrightarrow{\partial}}{\partial\tau_1} < q'_1,\ldots,out|\chi(\tau_1,\alpha)|q_2,\ldots,in>, \end{aligned} \tag{19}$$

which can be reduced to

$$\begin{aligned} &< q'_1,\ldots,out|q_1,\ldots,in>=< q'_1,\ldots,out|a^+_{out}|q_2,\ldots,in>+ \\ &i\int d\tau_1 d\alpha e^{-i\varepsilon\tau}Y_{lnm}(\alpha)\Big[\frac{\partial^2}{\partial\tau_1^2}+\hat{L}^2(\alpha)+1\Big] < q'_1,\ldots,out|\chi(\tau_1,\alpha)|q_2,\ldots,in>. \end{aligned} \tag{20}$$

The first term represents a disconnected part of amplitude. By repeating this reduction step by step we obtain the following result (apart from disconnected terms):

$$\begin{aligned} &< q'_1\ldots q'_k, out|q_1\ldots q_s in) = \\ &i^{k+s}\int d\tau_1 d\alpha_{\mathbf{1}}\ldots d\tau_s d\alpha_{\mathbf{s}} e^{(i\sum_{j=1}^k \varepsilon_j\tau_j - \sum_{j=1}^s \varepsilon_j\tau_j)}\prod_{j=1}^k Y_{l_jn_jm_j}(\alpha_{\mathbf{j}})\prod_{j'=1}^s Y^*_{l_{j'}n_{j'}m_{j'}}(\alpha'_{\mathbf{j}}) \\ &\times\hat{\mathbf{A}}(\tau_1,\alpha_{\mathbf{1}})\ldots\hat{\mathbf{A}}(\tau_s,\alpha_{\mathbf{s}}) < 0|R\chi(\tau_1\alpha_{\mathbf{1}})\ldots\chi(\tau_s\alpha_{\mathbf{s}})|0>. \end{aligned} \tag{21}$$

The symbol R expresses the fact, that the product of operators is ordered along the radius of sphere S^4. It will be mentioned, that on mass shell condition of conventional theory is replaced here by $\varepsilon^2 \to (l+1)^2$. Having this scheme we can consider the asymptotic behavior of Green function - to be more exact its (εlnm) transform - for large ε similar to the high energy behavior of the Fourier transform of the Green function, considered in[10], and show the importance of singular trajectories.

3. CLASSICAL SINGULAR TRAJECTORIES AND CROSS SECTION.

We shall follow to paper[13] in order to calculate semiclassically the total cross section induced by scattering of two particles, connecting it with the help of the optical theorem with the imaginary part of the diagonal matrix element of the scattering amplitude $< \varepsilon lnm|M|\varepsilon lnm >$:

$$\begin{aligned}\sigma(\varepsilon) \sim \quad & \lim_{\varepsilon^2 \to (l+1)^2} Im \int d\tau_1 ... d\tau_4 d\alpha_1 ... d\alpha_4 \exp\{-i\varepsilon_1(\tilde{\tau}_1 - \tilde{\tau}_3) - i\varepsilon_2(\tilde{\tau}_2 - \tilde{\tau}_4)\} \\ & \times Y^*_{lnm}(\alpha_1) Y^*_{lnm}(\alpha_2) Y_{lnm}(\alpha_3) Y_{lnm}(\alpha_4) < \hat{\mathbf{A}}_1 \chi(\tilde{\tau}_1, \alpha_1) ... \hat{\mathbf{A}}_4 \chi(\tilde{\tau}_4, \alpha_4) >_0, \quad (22)\end{aligned}$$

The expression (22), which is our starting formula, looks like the corresponding formula of conventional field theory with replacement $t \to \tilde{\tau},\ E \to \varepsilon,\ \vec{k} \to (lnm)$. The requirement of mass shell condition is replaced by $\varepsilon^2 \to (l+1)^2$. According to [8],[9],[12] we arrive at singular trajectories, parametrized by pure imaginary time $\tilde{\tau} = -i \ln r = -i\tau$.

In our field theory the trajectory begins at $\tau = -\infty$ from vacuum (we suggest that the potential $U(\chi)$ is double-well), where $\varepsilon = 0$ and goes to the singularity at some value of $\tau = -\tau_0/2$. At this point the dimensionality ε receives an increment and field proceeds further with the fixed ε to the first turning point at $\tau = 0$. At the turning point the field can enter in principle the region, where τ is real (it corresponds to the Minkowskian part of conventional theory). But in this region the exponential is pure phase. Next the amplitude can be squared. It means, that the trajectory has to be replaced in opposite direction going from turning point with fixed nonzero ε to the singularity then returning ultimately to the vacuum. Finally one can be obtained the following result for the cross section (up to the exponential accuracy):

$$\sigma(\varepsilon) = e^{-S(\varepsilon)} \quad (23)$$

where S is full classical action and

$$S = S^{\mathrm{I}} - S^{\mathrm{II}} - S^{\mathrm{III}} + S^{\mathrm{IV}} - S^{\mathrm{V}}. \quad (24)$$

Here $S^{\mathrm{I}} - S^{\mathrm{V}}$ are pieces of action calculated at different branches and are defined by:

$$\begin{aligned} S^{\mathrm{I}} &= S^{\mathrm{IV}} = \textstyle\int_0^\infty d\chi \sqrt{2U(\chi)}, \\ S^{\mathrm{II}} &= S^{\mathrm{III}} = \textstyle\int_{\chi_t}^\infty d\chi \sqrt{2(U(\chi) - \varepsilon)}, \\ S^{\mathrm{V}} &= \textstyle\int_{-\tilde{\chi}_t}^{\tilde{\chi}_t} d\sqrt{2(U(\chi) - \varepsilon)}. \end{aligned} \quad (25)$$

Clearly the branches $S^{\rm I}, S^{\rm IV}$ correspond to the part of trajectory with zero ε, while the branches $S^{\rm II}$,$S^{\rm III}$, $S^{\rm V}$- to those with nonzero ε. The branch $S^{\rm V}$becomes zero, if the dimensionality is higher then potential barrier. One can show, that each of $S^{\rm I} - S^{\rm IV}$ diverges, but the sum is finite.

4. THE MASSLESS SCALAR THEORY.

Consider the simplest example of scalar massless theory with the Euclidean action

$$S = \int d^4x \Big\{ \frac{1}{2} \partial_\mu \phi(x) \partial_\mu \phi(x) + \frac{g^2}{4} \phi^4(x) \Big\}. \tag{26}$$

In spherical coordinates it reads:

$$S = \int_0^\infty d(\ln r) \Big\{ \frac{1}{2} (\frac{\partial \chi(r,\alpha)}{\partial (\ln r)})^2 + \frac{1}{2} \chi(r,\alpha)(\hat{L}^2 + 1)\chi(r,\alpha) + \frac{g^2}{4} \chi^4(r,\alpha) \Big\} \tag{27}$$

with the effective potential

$$U_{eff}(\chi) = \frac{1}{2} \chi^2(r,\alpha) + \frac{g^2}{4} \chi^4(r,\alpha) \tag{28}$$

We are interested in singular field configurations, parametrized by real $\ln r$. Assuming the angular independence of field we get the following equation of motion:

$$\frac{d^2 \chi}{d(\ln r)^2} - \chi - g^2 \chi^3 = 0 \tag{29}$$

or

$$\dot{\chi}^2 = \frac{g^2}{2} \chi^4 + \chi^2 - 2\varepsilon \tag{30}$$

with ε as constant of integration. For branches I and IV ($\varepsilon = 0$) we obtain:

$$\chi^{\rm I}(\tau) = -\frac{\sqrt{2}}{g} \frac{1}{\sinh(\tau + \tau_0/2)}, \quad \text{for} \quad \tau < 0, \tag{31}$$

$$\chi^{\rm IV}(\tau) = \frac{\sqrt{2}}{g} \frac{1}{\sinh(\tau - \tau_0/2)}, \quad \text{for} \quad \tau > 0. \tag{32}$$

For nonzero ε the solutions are expressed in terms of Jacobian elliptic functions:

$$\chi^{\rm II}(\tau) = -\frac{1}{g} \sqrt{\frac{2\sqrt{1+4g^2\varepsilon}}{\mathrm{sn}^2[\sqrt[4]{1+4g^2\varepsilon}(\tau + \tau_0/2)]} - (1 + \sqrt{1+4g^2\varepsilon}} \text{for} \quad -\tau_0/2 < \tau < 0, \tag{33}$$

$$\chi^{\rm III}(\tau) = \frac{1}{g} \sqrt{\frac{2\sqrt{1+4g^2\varepsilon}}{\mathrm{sn}^2[\sqrt[4]{1+4g^2\varepsilon}(\tau - \tau_0/2)]} - (1 + \sqrt{1+4g^2\varepsilon}} \text{for} \quad \tau_0/2 > \tau > 0, \tag{34}$$

We see,that the solutions $\chi^{\mathrm{I}}(\tau)$ and $\chi^{\mathrm{II}}(\tau)$ are singular at the points $\tau = -\tau_0/2$ and $\chi^{\mathrm{III}}(\tau)$, $\chi^{\mathrm{IV}}(\tau)$ at $\tau = \tau_0/2$ and for $\varepsilon = 0$ solutions $\chi^{\mathrm{II,III}}(\tau)$ coincide with $\chi^{\mathrm{I,IV}}(\tau)$. The elliptic functions depend on parameter ε defined by

$$k = \frac{\sqrt{1+\sqrt{1+4g^2\varepsilon}}}{\sqrt{2}\sqrt[4]{1+4g^2\varepsilon}}. \tag{35}$$

Besides they are complex-valued doubly periodic functions. The turning points are defined by requirement $d\chi(\tau)/d\tau =)$, of which the solutions are:

$$\begin{gathered} \chi_t^{1,2} = \pm\sqrt{-1+\sqrt{1+4g^1\varepsilon}}, \\ \chi_t^{3,4} = 0. \end{gathered} \tag{36}$$

The last one coincides with the vacuum. So we have only two finite turning points at real τ. It means, that the branch S^{V} becomes zero and there is no tunneling in theory. Collecting all these results and inserting into (24)-(26) we obtain:

$$\frac{d\ln\sigma(\varepsilon)}{d\varepsilon} = -\frac{1}{\sqrt[4]{1+4g^2\varepsilon}}\mathbf{K}(k), \tag{37}$$

where $\mathbf{K}(k)$ is a complete elliptic function of first kind. Using the expansion of $K(k)$ in two limiting cases $\varepsilon \ll 1$ and $\varepsilon \gg 1$ and integrating over ε we get respectively:

$$\ln\sigma(\varepsilon) = -\frac{1}{2}\varepsilon(\ln\frac{16}{\varepsilon g^2}+1) + \frac{1}{16}\varepsilon^2 g^2(3\ln\frac{16}{\varepsilon g^2} - \frac{25}{2}) + O(\varepsilon^3) \quad \text{for} \quad \varepsilon \ll 1, \tag{38}$$

and

$$\ln\sigma(\varepsilon) = -\frac{[\Gamma(1/4)]^2}{3\sqrt[4]{4}\pi^2 g^2}\varepsilon^{3/4} + O(\varepsilon^{1/4}) \quad \text{for} \quad \varepsilon \gg 1. \tag{39}$$

We see, that the cross section decreases as a function of ε and reproduces the result of [13].

5. SU(2) YANG-MILLS THEORY.

The next model we consider is a pure Yang-Mills theory, the Euclidean action of which is

$$S = \frac{1}{4}\int dx^4 F^a_{\mu\nu}F^a_{\mu\nu} \tag{40}$$

with

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon_{abc}A^b_\mu A^c_\nu. \tag{41}$$

For zero energies the singular trajectories have been indicated by S.Yu. Khlebnikov[11]. They are BPST [15] instanton solutions with some modification. We derive below the singular trajectories for all values of ε as exact solutions of equation of motion, using the ansatz of BPST:

$$A^a_\mu = \frac{1}{g}\eta^a_{\mu\nu}n_\nu\frac{\phi(r)}{r}, \tag{42}$$

where n_ν is again a unit vector, parametrized by spherical coordinates and $\eta^a_{\mu\nu}$ are quantities introduced by t'Hooft[16]. On substituting this Ansatz into the action last one reduces to

$$S = \frac{3\pi^2}{g^2}\int d\tau\{\frac{1}{2}(\frac{d\phi}{d\tau})^2 + \frac{1}{2}(\phi^2 - 2\phi)^2\} \tag{43}$$

with the double-well potential:

$$U(\phi) = \frac{1}{2}(\phi^2 - 2\phi)^2.$$

which has two minima equal to zero at $\phi = 0, 2$ and one maximum equal to $1/2$ at $\phi = 1$. For $\phi(\tau)$ one obtains the equation:

$$\frac{d\phi}{d\tau}^2 = (\phi^2 - 2\phi)^2 - 2\varepsilon. \tag{44}$$

For $\varepsilon = 0$ we obtain:

$$\phi^{\mathrm{I}}(\tau) = 1 + \coth(\tau + \tau_0/2), \quad \text{for} \quad \tau < 0, \tag{45}$$

$$\phi^{\mathrm{IV}}(\tau) = 1 - \coth(\tau - \tau_0/2), \quad \text{for} \quad \tau > 0, \tag{46}$$

while for nonzero ε one gets two sets of solutions:
a)for $2\varepsilon < 1$:

$$\phi_0^{\mathrm{II}} = 1 + \frac{\sqrt{1+\sqrt{2\varepsilon}}}{\mathrm{sn}[\sqrt{1+\sqrt{2\varepsilon}}(\tau + \tau_0/2)]}, \quad \text{for} \quad \tau < 0, \tag{47}$$

$$\phi_0^{\mathrm{III}} = 1 - \frac{\sqrt{1+\sqrt{2\varepsilon}}}{\mathrm{sn}[\sqrt{1+\sqrt{2\varepsilon}}(\tau - \tau_0/2)]}, \quad \text{for} \quad \tau > 0, \tag{48}$$

b) b)for $2\varepsilon > 1$:

$$\phi^{\mathrm{I}} = 1 + \sqrt{1 - \sqrt{1+2\varepsilon} - \frac{2\sqrt{2\varepsilon}}{\mathrm{sn}^2\sqrt{2\sqrt{2\varepsilon}}(\tau + \tau_0/2)}}, \quad \text{for} \quad \tau < 0 \tag{49}$$

$$\phi^{\mathrm{IV}} = 1 - \sqrt{1 - \sqrt{1+2\varepsilon} - \frac{2\sqrt{2\varepsilon}}{\mathrm{sn}^2\sqrt{2\sqrt{2\varepsilon}}(\tau - \tau_0/2)}}, \quad \text{for} \quad \tau > 0 \tag{50}$$

It is easy to verify, that (47) and (48) coincide with (45), (46) for $2\varepsilon \to 0$. Besides for $2\varepsilon = 1$ (47) and (48) reduces to (49) and (50) correspondingly. At the turning points $d\phi(\tau)/d\tau = 0$. The solutions of this are:
a)for $2\varepsilon < 1$:

$$\phi_1^{\mathrm{II}} = 1 + \sqrt{1+\sqrt{2\varepsilon}}, \tag{51}$$

$$\phi_2^{\mathrm{III}} = 1 - \sqrt{1+\sqrt{2\varepsilon}}, \tag{52}$$

$$\phi_3^{\mathrm{II}} = 1 + \sqrt{1-\sqrt{2\varepsilon}}, \tag{53}$$

$$\phi_4^{\mathrm{III}} = 1 - \sqrt{1-\sqrt{2\varepsilon}}; \tag{54}$$

b)for $2\varepsilon > 1$:

$$\phi_1^{\mathrm{II}} = 1 + \sqrt{1 + \sqrt{2\varepsilon}}, \tag{55}$$

$$\phi_2^{\mathrm{III}} = 1 - \sqrt{1 + \sqrt{2\varepsilon}}, \tag{56}$$

$$\phi_3^{\mathrm{II}} = \phi_4^{\mathrm{III}} = 0; \tag{57}$$

We see from (55)-(57), that there is no tunneling in this case. This is not surprising, since the trajectory goes over the barrier. More interesting is the case $2\varepsilon < 1$, when the trajectory penetrates the barrier and the branch S^{V} contributes to the cross section. Taking derivative of $\ln\sigma(\varepsilon)$ with respect ε as in the massless scalar theory we obtain:
a)for $2\varepsilon < 1$:

$$\frac{d\ln\sigma(\varepsilon)}{d\varepsilon} = \frac{3\pi^2}{2g^2\sqrt{1+\sqrt{2\varepsilon}}}\mathbf{K}\left(\sqrt{\frac{1-\sqrt{2\varepsilon}}{1+\sqrt{2\varepsilon}}}\right); \tag{58}$$

b)for $2\varepsilon > 1$:

$$\frac{d\ln\sigma(\varepsilon)}{d\varepsilon} = -\frac{3\pi^2}{2g^2\sqrt{2\sqrt{2\varepsilon}}}\mathbf{K}\left(\sqrt{\frac{\sqrt{2\varepsilon}-1}{2\sqrt{2\varepsilon}}}\right) \tag{59}$$

Using well known expansions of complete elliptic functions in two limiting values of ε - $2\varepsilon \ll 1$ and $2\varepsilon \gg 1$ - one can obtain after integrating over ε the following expressions for the total cross section;
a)for $2\varepsilon \ll 1$:

$$\ln\sigma(\varepsilon) = \frac{3\pi^2}{g^2}\left\{\frac{1}{4}\varepsilon\left(\ln\frac{32}{\varepsilon}+1\right) - \frac{3}{128}\varepsilon^2\left(\ln\frac{32}{\varepsilon}-\frac{3}{2}\right)\right\} + O(\varepsilon^3); \tag{60}$$

b)for $2\varepsilon \gg 1$:

$$\ln\sigma(\varepsilon) = -\frac{3\pi^2[\Gamma(1/4)]^2}{3\sqrt[4]{8}g^2}\varepsilon^{3/4} + O(\varepsilon^{1/4}). \tag{61}$$

Analysis of the obtained results shows rising behavior of the total cross section for $2\varepsilon \ll 1$, whereas it decreases for $2\varepsilon \gg 1$. Evidently the cross section reaches its maximal value, which occurs at the value of 2ε, defined by the top of barrier - in particular at $\varepsilon = 1/2$. All these are in agreement with those of[13]. Summarize all these one should mention, that there are infinite many ways in field theory to reach the singularity. In our field theoretical approach this way is chosen naturally. S, the use of BPST's ansatz makes it possible to look for singular classical configurations without any further approximation.

The more realistic theory is studied and is the subject of other paper.

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